## APPENDIX E: HUYGHENS-FRAUNHOFER-KIRCHHOFF APPROXIMATION

We shall use the WKB (eikonal) approximation up to the exit surface of the lens, but construct a solution of the wave equation which is better than the WKB expression in the space beyond the lens. This requires input of just the WKB values for $U$ and $\nabla U$ values at the exit surface of the lens. The solution beyond the lens is provided by Green's theorem:

$$U(\mathbf{x}) = \frac{1}{4\pi}\int_{\Sigma_0} d\mathbf{\Sigma_0}\cdot[U(\mathbf{x_0})\nabla_0 G(\mathbf{x},\mathbf{x}_0) - G(\mathbf{x},\mathbf{x}_0)\nabla_0 U(\mathbf{x_0})]. \tag{E1}$$

In what follows, it is helpful to use symbols illustrated in Fig. (19), although the details associated with this particular lens are not needed. In Eq. (E1), $\mathbf{x} = \mathbf{r}$ is the observation point beyond the lens. $\mathbf{x}_0 = -L\hat{\mathbf{k}} + \mathbf{r}_1$ represents a point on $\Sigma_0$, the exit surface of the lens. (It also includes the screen, but on it $U$ and $\nabla U$ are taken to vanish). $d\mathbf{\Sigma}_0 = d\Sigma_0\hat{n}$ is the surface element of the lens, whose normal $\hat{n}$ points radially outward from it. $G(\mathbf{x},\mathbf{x}_0)$ is the Green's function for the vacuum, given by Eq. (C13) with $r = |\mathbf{x}-\mathbf{x}_0| \equiv D$. It satisfies the wave equation with a point source (Eq. (C1), with $n = 1$ and with the argument of the delta function changed to $\mathbf{D}$). Thus, Eq. (E1) describes $U$ as a continuous sum (integral) of solutions of the wave equation so, of course, it is a solution of the wave equation.

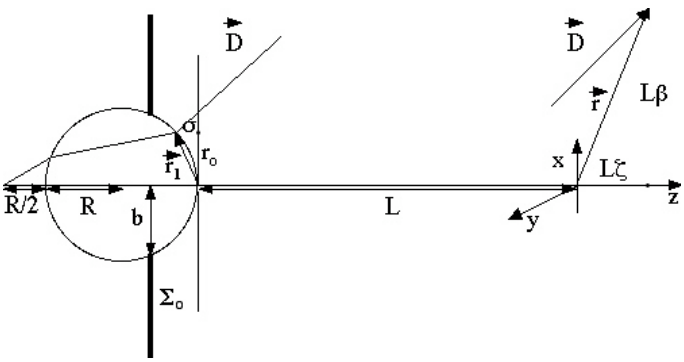


FIG. 19: Ray geometry for a ball lens

Eq. (E1) can be simplified. From (C13),

$$\nabla_0 G = -G\hat{\mathbf{D}}[ik - D^{-1}] \approx -G\hat{\mathbf{D}}ik,$$

where the approximation is valid for $D >> \lambda$. From (C12),

$$\nabla_0 U(\mathbf{x}_0) = ikG(\mathbf{x}_0)\nabla_0\Phi(\mathbf{x}_0) \approx ikG(\mathbf{x}_0)\hat{\mathbf{v}}_0,$$

where the approximation replaces $\Phi$ by $\Phi_0$ (since $\Phi_1$ is quite constant over the lens exit surface) and uses (C4). Thus, (E1) becomes:

$$U(\mathbf{x}) = \frac{-ik}{4\pi}\int_{\Sigma_0} d\Sigma_0 U(\mathbf{x_0})\frac{1}{D}e^{ikD(\mathbf{x},\mathbf{x}_0)}\hat{n}\cdot[\hat{\mathbf{v}}_0 + \hat{\mathbf{D}}].$$

We are interested in the solution for large L, on the image plane far from the lens. There, $D^{-1}$ varies slowly, and may be taken out of the integral.

As shown at the end of section (B 7), the outgoing ray from the lens surface satisfies is almost parallel to the $z$-axis (the optic axis), i.e., $\hat{\mathbf{v}}_0 \approx \hat{\mathbf{k}}$. (For a perfect lens, $\hat{\mathbf{v}}_0 = \hat{\mathbf{k}}$ since then the source point is imaged at $\infty$.) Similarly, $\hat{\mathbf{D}} \approx \hat{\mathbf{k}}$ since the intensity at $\mathbf{x}$ we wish to explore is not very much off-axis. The normal to the exit lens surface is not parallel to $\hat{\mathbf{k}}$, but $d\mathbf{\Sigma_0}\cdot\hat{\mathbf{k}} = d\Sigma_0\hat{n}\cdot\hat{\mathbf{k}} = dA_0$, where $dA_0$ is the surface element of $S_0$, the plane tangent to the exit surface of the lens at the point where it intersects the optic axis and perpendicular to the optic axis (the "tangent plane"). Therefore, the surface integral can be converted from being over the exit surface of the lens to being over the tangent plane. With $U(\mathbf{x_0}) \sim \exp ik\Phi_0(\mathbf{x_0})$ given by the WKB approximation, the approximate solution to be evaluated is

$$U(\mathbf{x}) \sim \int_{S_0} dA_0 e^{ik[\Phi_0(\mathbf{x_0})+|\mathbf{x}-\mathbf{x}_0|]}. \tag{E2}$$

Eq. (E2) is what we need hereafter. Since we are only interested in relative values of $|U(\mathbf{x})|^2$, constant factors may be dropped or chosen at pleasure.

It is worth re-emphasis, that $\Phi_0(\mathbf{x_0})$ in Eq. (E2) is the optical path length (C9), from the source to the exit surface of the lens, at height $r_0$ above the optic axis. It is *not* the optical path length from the source to the tangent plane whose surface area element is integrated over in Eq. (E2).

## APPENDIX F: POINT SPREAD FUNCTION AND CONSEQUENCES

### 1. The Diffraction Integral

To integrate (E2), we need $D = |\mathbf{x}-\mathbf{x}_0|$. Again, refer to Fig.(19). The origin of the coordinate system is on the optic axis, a large distance $L$ away from the exit surface of the lens. $\mathbf{D}$ makes a small angle $\beta$ with respect to the optic axis, and its horizontal component extends a small distance $\zeta L$ beyond the origin. Therefore, the observation point is

$$\mathbf{x} = \mathbf{r} = \hat{\mathbf{i}}L\beta + \hat{\mathbf{k}}L\zeta.$$

The point on the surface of the lens is

$$\mathbf{x}_0 = \hat{\mathbf{i}}r_0\cos\phi + \hat{\mathbf{j}}r_0\sin\phi - \hat{\mathbf{k}}[L+\sigma]$$

where $\phi$ is the azimuthal angle in the tangent plane and

$$\sigma = R - \sqrt{R^2 - r_0^2} \approx \frac{r_0^2}{2R^2} + \frac{r_0^4}{8R^3}$$